

REVIEW ARTICLE

Numerical relativity confronts compact neutron star binaries: a review and status report

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Abstract. We review the current status of attempts to numerically model the merger of neutron star-neutron star (NSNS) and black hole-neutron star (BHNS) binary systems, and we describe the understanding of such events that is emerging from these calculations. To accurately model the physics of NSNS and BHNS mergers is a difficult task. It requires solving Einstein's equations for dynamic spacetimes containing black holes. It also requires evolving the hot, supernuclear-density neutron star matter together with the magnetic and radiation fields that can influence the post-merger dynamics. Older studies concentrated on either one or the other of these challenges, but now efforts are being made to model both relativity and microphysics accurately together. These NSNS and BHNS simulations are then used to characterize the gravitational wave signals of such events and to address their potential for generating short-duration gamma ray bursts.

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Compact object binaries consisting of either two neutron stars (NSNS binaries) or a black hole (BH) and a neutron star (NS) (BHNS binaries) are unique laboratories for studying both strongly-curved spacetime and supernuclear-density matter. Such binaries radiate away orbital energy through gravitational waves, causing the two objects to slowly spiral in closer to one another and, eventually, to collide and merge.

Estimating the rate of NSNS/BHNS mergers is not easy. In our galaxy, there are several observed NSNS binaries that should merge due to gravitational radiation-induced inspiral in less than a Hubble time. (The double pulsar system J0737-3039 will coalesce in 85Myr [1].) Extrapolating from these observations, the rate of NSNS mergers per Milky Way equivalent galaxy has been estimated in the range 40-700 per Myr [2]. In contrast, no BHNS binaries have yet been detected. The merger rate can also be estimated theoretically. NSNS and BHNS binaries are thought to form from binary systems of two massive ($> 8M_{\odot}$) stars. Population synthesis calculations model the evolution of a large number of massive stellar binaries to determine the likelihood of producing tight NSNS or BHNS systems (see e.g. [3, 4] for details.). There are large uncertainties regarding several aspects of the evolution of these binaries, leading to more than an order of magnitude uncertainty in the predicted NSNS and BHNS merger rates. Nevertheless, the predicted NSNS merger rate is consistent with that inferred from the number of observed systems, and the BHNS rate is predicted to be about two orders of magnitude lower [5].

NSNS/BHNS mergers might have a number of observable effects. The most direct way to observe them is through the gravitational waves emitted during the late inspiral and merger. Population synthesis calculations predict that Advanced LIGO should observe of order 10 NSNS merger events per year, and about one BHNS event [5]. These events are also thought to be the most promising explanation of the majority of observed short-duration gamma ray bursts (GRBs). Most GRB models require a BH surrounded by a large accretion disk. Energy is extracted, either from the disk or the black hole's spin, and is dumped into a baryon-poor region around the poles. This drives the ultrarelativistic outflow needed to explain the observed athermal spectra of GRBs. (For a review of short GRB progenitor models, see [6].) Only numerical simulations can test the assumptions of this model: a massive post-merger accretion disk, a baryon-poor region, and efficient energy extraction through neutrinos or magnetic fields. Finally, NSNS/BHNS mergers may be important for understanding the observed abundances of the heavy elements that are formed by rapid neutron capture in the r-process [7]. (See [8] for a recent review of the r-process and its proposed sites.) Here again simulations are needed to determine how much and what kind of matter is ejected during such mergers.

In the next section, we review the physics and numerics involved in modeling NSNS/BHNS mergers. Then, for each type of binary, we describe the current understanding of the merger process derived from simulations. For another useful review of this field, see Faber [9].

1. The numerical challenge

Neutron stars have compactness in the range $GM/Rc^2 \approx 0.1 - 0.2$, so we expect general relativistic (GR) effects to be important to their evolution. Important studies have nevertheless been carried out using Newtonian physics (see below). For BHNS binaries, this means the BH is modeled as a Newtonian point mass. A next step towards GR is to modify the point mass potential to mimic certain GR effects, such as having an innermost stable circular orbit (ISCO) for test particle orbits [10, 11]. In many cases, a good approximation to full GR is the conformal flatness approximation (CFA) [12]. CFA assumes the spatial metric g_{ij} is conformally flat at all times: $g_{ij} = \psi^4 \delta_{ij}$. (Throughout this paper, Latin indices run from 1 to 3, while Greek indices run from 0 to 3, with 0 being the time index.) From this assumption, one can derive elliptic equations for ψ (from the Hamiltonian constraint), for the shift β^i (from the condition that g_{ij} remain conformally flat together with the momentum constraint), and for the lapse α (from the maximal slicing condition). CFA is always accurate to first PN order; for a spherically-symmetric system it is exact, because for such systems, there are always coordinates in which g_{ij} is conformally flat. For spinning NS, errors from assuming CFA are a few percent [13]. The spatial metric for a spinning Kerr BH cannot be made conformally flat (at least for a broad class of foliations) [14], and assuming CFA for spacetimes with rapidly-spinning BH has been found to introduce significant junk radiation [15, 16]. The only way to accurately evolve general BH spacetimes is using full GR. Another limitation of Newtonian and CFA calculations is that gravitational waves must be estimated using

the quadrupole approximation, while in GR simulations the waves come directly from the spacetime evolution.

The first challenge for a full GR evolution is to acquire valid initial data. This must both satisfy the ADM constraint equations and represent an astrophysically reasonable late-inspiral binary configuration. During the inspiral, the infall timescale is much longer than the orbital period. Also, gravitational radiation will damp away any orbital eccentricity before the separation becomes small. The exception might be mergers in dense stellar clusters, where 3-body interactions can be important, but these are expected to be a minority of NSNS and BHNS binaries [17, 18]. (This expectation has recently been challenged, however [19].) Therefore, circular orbit is usually taken to be a good first approximation for the late inspiral. Combining the circular orbit assumption with CFA, one can construct quasi-equilibrium profiles of the spacetime and the NS fluid. By computing these profiles for many binary separations, one can construct a sequence of “snapshots” that tell the history of the binary’s inspiral. [To make each snapshot correspond to the same binary, the baryon mass of the NS(s) and irreducible mass of the BH are fixed.] During the late inspiral, the quasi-equilibrium approximation breaks down and full time evolutions are needed. A quasi-equilibrium snapshot of the late inspiral is then used as initial data for the evolution. Neglecting the initial radial (i.e. infall) velocity is known to produce spurious eccentricity in the orbit, but this can be removed by adding back an initial infall [20, 21, 22]. CFA produces an initial burst of spurious “junk” gravitational radiation. Junk radiation can make it particularly difficult to create initial data with a rapidly spinning BH. This difficulty can be reduced by using a Kerr-Schild background metric, rather than CFA [23, 21]. Some junk radiation will still be present, however, because the chosen conformal spatial metric still does not correspond to that of a true inspiraling binary.

Evolving the metric forward in time requires choosing a formulation of GR and choosing a gauge. There are currently two formulations of GR used for NSNS/BHNS simulations: Baumgarte-Shapiro-Shibata-Nakamura (BSSN) [24, 25] and generalized harmonic (GH) [26, 27, 28, 29]. In both formulations, the “trick” is to promote certain first spatial derivatives of the metric to the status of independent variables; this turns the evolution equations for metric components into wave-like equations. The gauge conditions fix the evolution of the coordinates. For BSSN, this is done by choosing the lapse α and shift vector β^i evolution equations; the “1+log” lapse and “Gamma-driver” shift have proven immensely successful. With these gauges, BSSN codes can stably evolve spacetimes with BH singularities inside the computational domain [30, 31]. This treatment of BH spacetimes is called the “moving puncture” approach because BHs can freely move through the grid, a feature which has made it possible to do long-term, stable evolutions of binary black hole mergers with BSSN [32, 33]. For GH, the gauge is specified through the gauge source functions $H_\alpha = -g^{\beta\gamma}\Gamma_{\alpha\beta\gamma}$. Suitable choices for H_α are still under development (e.g. [34, 35]), and the moving puncture approach has not yet been implemented in GH evolutions. Instead, BH singularities are excised in these simulations, i.e. a region inside the horizon is cut out of the grid and replaced with an inner boundary. Pretorius has successfully simulated binary black hole mergers using GH with excision [36, 34]. The Caltech-Cornell-CITA group has also successfully used GH to

simulate a variety of binary BH mergers using high-accuracy spectral techniques [37, 38].

The state of the neutron star fluid at each point is given by six numbers: the baryon density ρ , temperature T , 3-velocity v^i , and electron fraction Y_e . The fluid is evolved using six conservation equations: baryon number conservation $(\rho u^\mu)_{;\mu} = 0$, energy-momentum conservation $T^\mu_{\nu;\mu} = 0$, and lepton number conservation $(\rho Y_e u^\mu)_{;\mu} = S_L$, where the lepton source term S_L comes from neutrino emission or absorption.

The stress tensor T^μ_ν depends on the pressure P and specific enthalpy h ; these are determined by the equation of state (EoS), which gives P and h as functions of ρ , T , and Y_e . The actual EoS of NS matter is not known, so several guesses and approximations are used for NSNS/BHNS simulations. (For reviews of the NS EoS and its astrophysical implications, see [39, 40].) One is to treat the NS as a polytrope: $P = \kappa \rho^{1+1/n}$, where the polytropic index n is a constant. This EoS is not temperature-dependent, so evolutions with this EoS will not allow the NS to heat. A more common EoS choice is to use the polytropic law to set the initial data for a cold NS and then evolve using the Γ -law $P = (\Gamma - 1)\rho\epsilon$, where ϵ is the specific internal energy and $\Gamma = 1 + 1/n$. The Γ -law is equivalent to the polytropic law as long as the fluid is continuous and isentropic, but it will allow heating when shocks are formed. Polytropic evolutions are still used to help gauge the importance of heating effects (e.g. [41]). A step towards more sophisticated EoS is to break up the pressure and internal energy into zero-temperature and thermal components, e.g. $P = P_{\text{cold}}(\rho) + P_{\text{thermal}}(\rho, T)$. Then the thermal part is set to a Γ -law [$P_{\text{thermal}} = (\Gamma - 1)\rho\epsilon_{\text{thermal}}$], while the cold part can be a complicated function of ρ . Recently, piecewise-polytropic laws for the cold EoS have been proposed [42] and used [43]. With only four parameters, this class of EoS can approximate any of the NS EoS proposed to date. It thus provides a good systematic way to cover the space of possible EoS. Other simulations use cold EoS based on nuclear physics calculations. Shibata and collaborators [44, 45, 22] have evolved NSNS binaries in GR using the FPS [46], SLy [47], and APR [48] cold EoS. The most realistic nuclear theory-based EoS have a general temperature and composition dependence. The tabulated Lattimer-Swesty [49] and Shen [50, 51] EoS have been used in Newtonian and CFA NSNS/BHNS simulations. Correct temperature dependence will probably be important for modeling mergers but not inspirals. Oechslin, Janka, and Marek [52] have compared many of these EoS in the context of NSNS mergers.

The transport of energy and lepton number by neutrinos can have important effects on the post-merger system. A few Newtonian NSNS/BHNS merger calculations have included these effects in approximate ways. The simplest way is through a leakage scheme [53, 54]. These model neutrino cooling by removing energy from each gridpoint and adjusting Y_e at a rate set to be some function of the fluid variables and the ν -optical depth τ_ν . This function is chosen to reproduce rates given by local reaction timescales in ν -transparent regions, while it reproduces rates given by the diffusion timescale in ν -opaque regions. Note that the scheme is local: ν 's emitted in one part of the grid can't be absorbed in another part, a fundamental limitation of leakage schemes. Also, one must somehow estimate τ_ν . A better but more complicated neutrino scheme, used in one study of the NSNS post-merger system [55] is flux-limited diffusion [56, 57]. In this case, the neutrino radiation intensity is evolved via a diffusion

equation, with fluxes limited so that free streaming is recovered in the ν -transparent regime. Even these simulations do not solve the full multi-angle Boltzmann transport equation, a task beyond current numerical resources. Much less work has been done on ν -transport in GR. De Villiers [58] and the Illinois (UIUC) group [59] have independently developed GR radiation transport schemes for the optically-thick limit, but these codes have not yet been applied to NSNS/BHNS binaries.

Recently, some NSNS merger simulations have included magnetic fields [60, 61, 62, 63]. These simulations all work in the ideal MHD limit—a good approximation in most cases, given the high conductivities inside NSs. To investigate cases where resistive effects are important (in low-density, high-temperature regions), Palenzuela *et al* [64] have developed, but not yet applied, a code to evolve the relativistic resistive MHD equations.

Modeling the important physical effects is only one aspect of NSNS/BHNS simulations. There is also the computational challenge of adequately resolving the several relevant length scales of the problem: the gravitational wavelength, the stellar radius, and the length scales of various fluid and MHD instabilities. Some simulations (e.g. [65, 66, 67, 68, 69, 52, 70]) use Smoothed Particle Hydrodynamics (SPH). The representation of the fluid by particles can naturally adapt to changes in the fluid’s size and shape. SPH has not yet been implemented in full GR. Most codes use grids to represent the fluid and metric. The problem of multiple scales is sometimes handled using Berger-Oliger moving-box-in-box adaptive mesh refinement (AMR). This technique is used by the Carpet module of the Cactus code [71], which is used in the UIUC [72] and Whisky [73] GRMHD codes. AMR is also used in the SACRA [74] code and by the BYU/LSU/LIU group’s code [75]. Testing is also being done on the use of multipatch (“cubed spheres”) grids in place of Cartesian boxes [76, 77, 78, 79]. Multipatch grids allow one to extend the grid outer boundary R at a cost proportional to R rather than R^3 . Finally, the Caltech-Cornell-CITA (CCC) group uses a two-grid technique, inspired by the CoCoNuT stellar collapse code [80]. The CCC code SpEC evolves the metric $g_{\mu\nu}$ pseudospectrally on one grid while evolving the fluid using standard finite-volume shock-capturing techniques on a second grid [81]. This code’s main advantage is that the fluid grid needs only extend as far as the NS matter, rather than out to the gravitational wave zone.

A final challenge in NSNS/BHNS modeling is that of covering the entire physically interesting range of binary parameters. NS masses may vary by tens of percents, while BH masses can vary widely. In addition, the BH can have a spin, with Kerr spin parameter a/M_{BH} anywhere from zero to one, oriented in any direction. The NS can also have a spin. However, the NS viscosity is thought to be too low for tidal effects to hold the NS in corotation [82, 83], so by the late inspiral, the orbital motion should dominate over the NS spin in most cases. Therefore, many simulations assume the NS to be initially irrotational. The effects of non-zero temperature (from tidal heating) [82, 84] and magnetic fields [62, 63] are expected to be negligible before the merger, and the pre-merger composition of the NS is fixed by β -equilibrium. Therefore, the main physical parameters to be varied are the masses and (for BHNS binaries) the BH spin. Also, the EoS, since it is not known, must be treated as another parameter (or, rather, set of parameters) to be varied.

2. Neutron star-neutron star mergers

2.1. History

The first numerical simulations of NSNS mergers used Newtonian physics and polytropic EoS [85, 86, 87, 65, 66, 88]. These studies found that the binary merges into a massive star rotating rapidly and differentially, but, being Newtonian, they could not check for the possibility that this object collapses to a BH. Simulations using CFA gravity and the zero-temperature Mayle-Wilson EoS [89] were carried out by Wilson, Mathews, and Marronetti [90, 12]. These pioneering computations surprisingly predicted that the NSs can collapse individually to BHs before merging. However, they used an EoS with fairly low NS maximum mass, and they were plagued by an error in one of their equations [91], so individual NS collapse is no longer considered likely [92].

After this, the field bifurcated—some groups concentrated on improving microphysics while retaining Newtonian gravity, and some groups concentrated on improving the treatment of gravity while retaining simplified microphysics. In the first category would be the simulations of Ruffert, Janka, and collaborators [53, 93, 54] and those of Rosswog and collaborators [68, 54, 94]. These simulations used the finite-temperature EoS of Lattimer-Swesty or Shen, and they used leakage models of neutrino cooling. Meanwhile, binary polytropes were evolved in CFA gravity by Oechslin, Rosswog, and Thielemann [95] and by Faber, Grandclement, and Rasio [96]. For full GR models, an important step was the development of accurate quasi-equilibrium configurations to serve as initial data [97, 98, 99, 100, 101]. Finally, the first fully GR simulations of NSNS merger were performed by Shibata and collaborators [102, 103, 104]. Since this time, efforts have been made to combine relativistic gravity with realistic microphysics. Oechslin, Janka, and Marek [52, 105] used realistic NS EoS in CFA simulations, while Shibata, Taniguchi, and Uryu [44, 45] modeled mergers in full GR using realistic zero-temperature EoS (plus Γ -law thermal components). In the past few years, magnetic field evolution has been added to both Newtonian [60] and GR [61, 62, 63] NSNS simulations. Also, the accuracy of these simulations has been greatly improved through the use of AMR [106, 41, 74]. Finally, efforts have been made to study the effects of MHD instabilities [107, 108] and neutrino energy transport [109, 110, 111, 55] on the post-merger system. The numerical studies to date can be combined to form a coherent picture, our current best guess, of what happens when two neutron stars collide.

2.2. The emerging picture

When two NS merge, they form a massive remnant. The first question to answer is “does the remnant collapse to a black hole?” It might seem like it should. There is a maximum mass $M_{\text{TOV,max}}$ for nonrotating NS, above which the star is dynamically unstable. $M_{\text{TOV,max}}$ depends on the unknown EoS; predictions of its value fall in the range $1.5\text{--}2.5M_{\odot}$ [39]. Two $1.4M_{\odot}$ stars will merge into a roughly $2.8M_{\odot}$ object. This is significantly above even high predictions of $M_{\text{TOV,max}}$, so collapse seems likely. However, rotation and, to a lesser extent, shock heating increase the maximum mass. A star uniformly rotating at its mass-shedding

limit (above which matter would be ejected from the equator) can be stable with a mass up to $M_{\text{sup}} \approx 1.2M_{\text{TOV,max}}$, due to centrifugal support [112]. Rigidly rotating stars with $M_{\text{TOV,max}} < M < M_{\text{sup}}$ are sometimes called supramassive. Greater centrifugal support, and hence greater maximum mass, can be achieved if the star rotates differentially, with the center having a significantly higher angular speed Ω than the equator [113, 114]. A NS supported above M_{sup} by differential rotation is called a hypermassive neutron star (HMNS). Numerical experiments in GR have demonstrated that HMNS are stable on dynamical timescales [115]. NSNS remnants are found by simulations to have angular speeds several times higher at the center than near the equator [102], so they are good HMNS candidates.

GR simulations find that, after the merger, a NSNS remnant may either collapse promptly (i.e. on a dynamical timescale) to a BH, or it may survive as a HMNS. The determining factor is the mass of the NSNS system M_{NSNS} . If M_{NSNS} is above a threshold mass M_{th} , the remnant collapses promptly; if $M_{\text{NSNS}} < M_{\text{th}}$, it forms a HMNS. All of the GR codes are in agreement on this basic point [102, 104, 45, 41, 72]. For $\Gamma = 2$ polytropes, $M_{\text{th}} \approx 1.7M_{\text{TOV,max}}$ [104], but for more realistic, stiffer EoS, M_{th} is found to be about $1.3 - 1.35M_{\text{TOV,max}}$ [45]. For $M_{\text{TOV,max}} = 2.1M_{\odot}$, the threshold is $M_{\text{th}} = 2.7 - 2.8M_{\odot}$. This is a typical NSNS binary mass, so it is quite possible that both prompt collapse and HMNS formation occur in nature.

The prompt collapse case can be a viable GRB engine only if the post-collapse BH is surrounded by a massive accretion disk. For equal-mass binaries, the disk mass is usually very small ($\ll 0.01M_{\odot}$) and the disk is very thin [45], making these systems poor candidates for producing GRBs. On the other hand, simulations find that the merger of unequal mass binaries proceeds much differently and can produce large disks [65, 116, 104, 45, 22]. In unequal mass binaries, the lower-mass NS can fill its Roche lobe before merger, so that it sheds matter both inward onto the more massive NS and outward into a tidal tail. The tail then falls back and contributes to the accretion disk around the BH. For a binary with mass ratio around 3:4, the disk can be $0.01M_{\odot}$ or larger, an excellent setup for a GRB [45]. In addition to the mass ratio, the disk mass also depends on the mass of the system—binaries with M_{NSNS} slightly above M_{th} produce more massive disks than those produced by more massive binaries [104, 22].

For M_{NSNS} close to M_{th} , the remnant survives as a HMNS, but only for a few milliseconds. This case was mentioned by Shibata, Taniguchi, and Uryu [44] and studied in detail by Baiotti, Giacomazzo, and Rezzolla [41]. In the latter’s simulation, the NSs merge, but then a bar-mode instability causes the two cores to split. The cores merge and split four times over a time of 8ms. During this time, the remnant is highly dynamical and asymmetric, and so it radiates gravitational waves. This radiation carries away angular momentum, so that the remnant loses centrifugal support and collapses, leaving a BH surrounded by a massive ($0.07M_{\odot}$) disk.

For lower-mass systems, the product of the merger is a HMNS. Such objects are dynamically stable, but they are vulnerable on longer timescales to processes (like gravitational radiation or magnetic fields) that sap the centrifugal support in the remnant’s core. If the star is axisymmetric, its rotation will not cause it to emit gravitational waves. However, for rapidly rotating stars with stiff EoS, the minimum energy configuration will be

ellipsoidal rather than spheroidal [117]. (A famous example would be the Maclaurin spheroid and Jacobi ellipsoid configurations for rapidly rotating incompressible stars [118].) Stiff stars tend to take ellipsoidal shape if $\beta = T/|W|$, the ratio of rotational kinetic to gravitational potential energy, exceeds a critical value. Realistic NS EoS tend to be stiff ($\Gamma \approx 2.75$) at high densities, and numerical simulations with Newtonian [65, 119], post-Newtonian [120] and full GR [44, 41, 43] physics have indeed found that NSNS remnants with these EoS usually form ellipsoids. These bar-like deviations from axisymmetry are stronger for stiffer assumed EoS [52]. Such stars emit strong gravitational waves and lose angular momentum J . This might have the effect of causing the star to contract or of lowering the ellipticity, but simulations indicate that the former effect dominates, and the remnant remains ellipsoidal as it contracts. Eventually, the remnant will reach a critical state and collapse to a BH. From the J -loss rate, the time for this to happen is inferred to be 30-100ms. Such long timescales are difficult to simulate in 3D with current resources, so the first simulations stopped well before the collapse. Oechslin and Janka [105] and Shibata and Taniguchi [45] estimated the mass of the disk around the post-collapse black hole based on the J distribution in the HMNS. Matter with specific angular momentum high enough for orbit outside the ISCO of the (not yet formed) BH was assumed to form the accretion disk. These calculations give disk masses of $\sim 0.1M_\odot$, high enough for GRBs. However, these estimates necessarily could not account for changes in the HMNS's J distribution prior to collapse. Finally, Baiotti, Giacomazzo, and Rezzolla [41] carried out long-time (~ 30 ms) simulations using a cold polytrope EoS that, for the first time, evolved the NSNS remnant to the point of delayed collapse (in their case, 16ms after merger). They found a final disk mass of $\approx 0.08M_\odot$, quite close to the earlier estimates.

Prompt collapse or ellipsoidal HMNS formation occur in most simulations, but other evolutionary paths are possible. If β of the remnant is below the critical value, the HMNS may form a spheroid which will eventually be driven to collapse by magnetic-driven, rather than gravitational wave-driven, processes (see below). This might happen for M_{NSNS} near but below M_{th} for some EoS [45]. Also, if $M_{\text{NSNS}} < M_{\text{sup}}$, the remnant might not collapse at all.

Shocks during the merger heat the remnant to temperatures of order 10 MeV. At these densities and temperatures, the remnant will radiate primarily in neutrinos. The HMNS has ν -optical depth of $10^2 - 10^4$, depending on the neutrino energy, and it will cool from radiation on a timescale of order 1 sec [55]. The neutrino emission from NSNS remnants has been studied using leakage schemes by Ruffert, Janka, and collaborators [53, 93, 121] and by Rosswog and Liebendörfer [54]. More recently, Dessart *et al* [55] have modeled neutrino transport in the HMNS under the assumption of axisymmetry using the multi-group flux-limited-diffusion code VULCAN/2D. These three codes are able to extract the neutrino signal from NSNS coalescence. They all find that the $\bar{\nu}_e$ emission dominates over the ν_e emission, an unsurprising result given the low Y_e of the remnant. Dessart *et al* also find high ν_μ/ν_τ luminosities. $\nu_i\bar{\nu}_i$ annihilation releases some $10^{49} - 10^{50} \text{ erg s}^{-1}$ outside the remnant. However, neutrino heating also drives a wind from the HMNS surface. This wind dumps $\sim 10^{-3}M_\odot \text{ s}^{-1}$ of matter into the polar regions, enough to baryon-load the poles and prevent a GRB from being produced while the HMNS remains [55]. Dessart *et al* speculate that a similar wind may be produced by the disk after the HMNS collapses, but in this case a centrifugal barrier

might keep the polar regions baryon-clean, allowing a GRB. The ν -driven wind also carries away some $10^{-3}M_{\odot}$ of neutron-rich material out of the HMNS system. GR may modify the above results, given that GR simulations tend to produce less massive tori around the HMNS than their Newtonian equivalents, and these tori contribute disproportionately to the neutrino emission [54].

Newtonian [122, 121] and CFA [52] simulations predict that $10^{-3} - 10^{-2}M_{\odot}$, depending on the EoS, of nuclear matter is ejected during NSNS mergers. The CFA simulations found two components to the ejecta: cold matter coming off the tidal tail and hot matter ejected from the surface of contact between the stars. As the expelled matter decompresses, neutrons are captured onto the heavy nuclei. The resulting nucleosynthesis has been modeled, with various approximations concerning the expansion rate and the initial composition and temperature, using r -process network calculations [123, 124]. For suitable parameters, the calculated abundances agree quite well with solar abundance pattern for mass numbers $A > 140$, especially regarding the $A = 195$ peak. However, reliable estimates of the ejecta mass and composition require GR simulations with realistic EoS.

The effect of the magnetic field in a NSNS binary is probably negligible prior to the merger for realistic field strengths [62, 63]. For very large fields ($\geq 10^{16}$ G), magnetic tension can suppress tidal deformation of the NSs and delay the merger [61, 63]. When the stars touch, a shear layer is formed at their interface. The shear layer is Kelvin-Helmholtz unstable, causing the flow to curl up into vortex rolls. Inside these vortex rolls, the magnetic field can be quickly amplified by orders of magnitude. This process was first modeled using Newtonian MHD by Price and Rosswog [60]. From a 10^{12} G pre-merger field in each NS, they found that the field in vortex rolls reached 10^{15} G in 1ms. They speculate that the field will reach equipartition in small vortices. These high-field pockets of matter will become buoyant, float up, and produce relativistic blasts when they hit the NS surface. If so, these blasts might help provide the energy for a GRB, and magnetic pressure might help deflect the neutrino-driven baryon wind. Kelvin-Helmholtz-driven field amplification has also been seen in GR simulations [61, 63]. In the simulations of Giacomazzo *et al* [63], the magnetic field saturates at more modest values (e.g. a 10^{12} G field grows by less than an order of magnitude). The discrepancy with the result of Price and Rosswog reflects the difficulty in numerically capturing the smaller-scale effects introduced by the Kelvin-Helmholtz instability.

Because the angular speed of the HMNS decreases with radius, it is unstable to the magnetorotational instability (MRI) [125]. The MRI produces small-scale turbulence that redistributes angular momentum outward, causing the core to slow, contract, and eventually collapse. This process has been modeled in GR MHD under the assumption of axisymmetry by a collaboration of the UIUC group and Shibata [107, 108, 126]. They found that the HMNS collapse leaves a BH surrounded by a massive accretion disk and baryon-clear poles. MRI-induced delayed collapse is likely the fate of spheroidal HMNS and ellipsoidal HMNS with very strong magnetic field.

Much of the interest in NSNS binaries is motivated by the prospect of extracting information about NS structure from inspiral and merger gravitational waveforms. The inspiral waveform may be particularly important for these purposes, because it includes

the frequencies at which Advanced LIGO will be most sensitive. The early, low-frequency inspiral waveform can be used to constrain the NS tidal Love number [127]. Faber *et al* [128] point out that the energy spectrum of the late inspiral waveform can constrain the NS radius. Read *et al* [43] have looked for EoS signatures in the late inspiral waveform by performing GR NSNS simulations and systematically varying the EoS. They conclude that the NS radius can be determined from the waveform to an accuracy of ~ 1 km for an event at 100 Mpc.

The merger, HMNS, and BH ringdown waveforms all have frequencies above 1kHz and so are more difficult to detect, but the mass and EoS-related differences become quite dramatic in them. If $M_{\text{NSNS}} > M_{\text{th}}$, the inspiral signal is followed by a merger waveform at frequencies $1\text{kHz} \lesssim f \lesssim 3\text{kHz}$, which in turn is followed by a (probably undetectable) BH ringdown signal at 6.5-7kHz [22]. If $M_{\text{NSNS}} > M_{\text{th}}$, the spectrum has another peak at $\sim 3\text{kHz}$ due to the radiation from the ellipsoidal remnant. A HMNS lasting $\sim 50\text{ms}$ located $\sim 50\text{Mpc}$ away could be detected by Advanced LIGO with a signal-to-noise ratio of around 3 [129]. On its own, this would be a weak signal, but it will be preceded by a more easily-detectable inspiral signal. The EoS strongly affects the compaction of the HMNS, and therefore its moment of inertia and its rotation frequency. Therefore, the frequency of the post-merger signal peak contains information on the EoS, particularly if M_{NSNS} can be extracted from the inspiral signal [129, 130].

3. Black hole-neutron star mergers

3.1. History

The first simulations of BHNS mergers were carried out a decade ago by Lee and Kluzniak [67, 131, 132, 133]. They used Newtonian physics, with the NS modeled as a polytrope and the BH modeled as a point mass. Next, simulations in Newtonian physics with nuclear-theory EoS were performed [134, 135], and the Newtonian potential was replaced by a Paczyński-Wiita potential [136, 137]. Faber *et al* [69] took a step toward GR by simulating extreme mass-ratio binaries using CFA physics. Löffler, Rezzolla, and Ansorg [138] modeled a head-on collision between a black hole and a neutron star in full GR. GR simulations of orbiting binaries became possible after quasi-equilibrium initial data was successfully generated [139, 140, 141, 142, 143, 21]. The first full-GR simulations of BHNS mergers were carried out by Shibata and Uryu [144, 145], followed quickly by the UIUC [146] and CCC [81] groups. These simulations all used polytropic EoS and only considered the case of zero initial BH spin. Meanwhile, Rantsiou *et al* [70] modeled BHNS mergers with spinning BH using a Kerr spacetime and an approximate treatment for the NS self-gravity. Their findings suggesting the importance of BH spin were confirmed when the UIUC group simulated the merger of several BHNS systems with nonzero initial BH spin in full GR [72].

3.2. The emerging picture

The mass of the black hole, M_{BH} , can vary widely from one BHNS system to another. For very high M_{BH} , the NS behaves like a test particle: it inspirals slowly until it reaches the ISCO

separation d_{ISCO} , at which point it plunges into the BH and is swallowed whole. For M_{BH} of only a few times the neutron star mass M_{NS} , the NS will overflow its Roche lobe at a radius d_{disr} before a plunge takes place. In this case, which resembles the unequal-mass NSNS case, the NS is tidally disrupted while outside the BH. Matter flows into the hole and outward into an extended tidal tail. Thus, we expect tidal disruption if $d_{\text{disr}} > d_{\text{ISCO}}$, and a plunge into the BH otherwise. In addition to the mass ratio, d_{disr} depends on the NS radius R_{NS} . A more compact star can survive stronger tidal forces and will disrupt closer to the BH. For expected R_{NS} , the critical mass ratio is about 4:1 [142]. For higher M_{BH} or smaller R_{NS} , one would expect the NS to be swallowed whole; for lower M_{BH} or larger R_{NS} , tidal disruption and mass transfer are expected.

The latest GR simulations by Shibata *et al* [147] confirm these expectations. Using a $\Gamma = 2$ polytropic EoS and fixing the initial BH spin to zero, they vary the mass ratio over the range 1.5:1 to 5:1, and they vary R_{NS} over the range 11-13km. For mass ratio 5:1, the NS is swallowed whole, leaving no disk. The gravitational waveform contains inspiral, merger, and BH ringdown signals which are similar to a binary BH merger of the same masses. The final kick velocity is also similar to the binary BH case. For mass ratios below 3:1, the NS disrupts before falling into the hole. The disruption spreads out the NS matter, so that the matter is more symmetrically distributed around the BH as it accretes, and the gravitational wave emission of the system greatly diminishes. This manifests itself in the Fourier spectrum of the waveform as a steep decline in amplitude above a cutoff frequency $f_{\text{cut}} \approx 1.3f_{\text{tidal}}$, where f_{tidal} is the wave frequency at d_{disr} . Since d_{disr} depends on R_{NS} , f_{cut} contains EoS information. If the star disrupts during its plunge phase, the waveform will have inspiral and merger signals, but the ringdown will be significantly weakened. If the the NS disrupts during the inspiral, then the merger part of the waveform is also suppressed. Disruption of the NS outside the BH also results in much lower kick velocities for the final hole than occur in the equivalent binary BH cases. Shibata *et al* [147], Etienne *et al* [146], and Duez *et al* [81] all find that, under the assumption of $\Gamma = 2$ EoS and nonspinning BH, BHNS mergers usually produce small accretion disks. Disk masses of $\sim 10^{-2}M_{\odot}$ were obtained by Shibata *et al* [147] for very low mass ratios (e.g. 2:1) and large NS radii ($\approx 14\text{km}$) [147]; in all other cases, the disk masses were much lower. In their most recent paper [72], the UIUC group find somewhat higher disk masses: they find a $0.06M_{\odot}$ disk for a case with a 3:1 mass ratio, $R_{\text{NS}} \approx 14\text{km}$, and no initial BH spin, a case that had not produced a significant disk in previous studies [146, 147]. Sources of inaccuracy in these studies include limited resolution and inadequate treatment of low-density regions. (See [72] for a discussion of the latter issue.)

Things are different when the BH is spinning, because d_{ISCO} for prograde orbits around a spinning Kerr BH is smaller than d_{ISCO} for a nonspinning hole of the same mass. Therefore, a NS would be expected to get closer to the BH and experience greater tidal forces before plunging. Etienne *et al* [72] have recently simulated several BHNS systems with nonzero BH spin aligned with the orbital rotation axis. For a mass ratio of 3:1 and a $\Gamma = 2$ EoS, they consider an aligned spin case with $a/M_{\text{BH}} = 0.75$ and an anti-aligned spin case with $a/M_{\text{BH}} = 0.5$. They find that the binary with aligned BH spin merges to produce a disk with mass $0.2M_{\odot}$ and temperature $\sim 4\text{MeV}$, a promising setup for a GRB. Aligned BH spin may

also make tidal disruption possible in cases where the BH is much more massive than the NS. An affine quasi-equilibrium model by Ferrari, Gualtieri, and Pannarale [148] predicts that, for a BH spin of $a/M_{\text{BH}} = 0.9$, NS will be disrupted for mass ratios up to 10:1 to 20:1, depending on the NS mass and EoS.

The merger can be strongly affected by the NS EoS. The EoS determines R_{NS} , the effects of which have already been discussed. The EoS also determines how R_{NS} changes with the NS mass, which can affect the character of the mass transfer. If a small transfer of mass leads the star to expand relative to its Roche lobe, mass transfer will be unstable. If the star contracts into its Roche lobe, mass transfer might be stable or turn on and off. In general, mass transfer tends to be more stable for stiffer EoS (higher Γ) because this corresponds to a greater tendency for a star to shrink when it loses mass [149].

Attempts have been made to estimate the effects of NS EoS in the context of BHNS simulations using Newtonian gravity. Lee and Kluzniak [67, 131] and Lee [132, 133] considered polytropes with Γ between $5/3$ and 3 . Janka, *et al* [134] used the Lattimer-Swesty EoS [49], while Rosswog, Speith, and Wynn [135] used the Shen EoS [50, 51]. These simulations showed large qualitative differences for different EoS assumptions. For Lattimer-Swesty nuclear matter, the NS disrupts in one mass transfer event, and a large post-merger disk is created. For Shen nuclear matter, a NS core can survive multiple mass transfer events, and the postmerger disk is much smaller. There are indications that the mass transfer is much less stable in GR, and the differences between the cases of stiff and soft EoS is not nearly so dramatic. The use of GR-mimicking potentials [10, 11] tends to eliminate episodic mass transfer [136, 137]. Simulations of large mass-ratio cases using CFA also find generally unstable mass transfer [69]. Most recently, the CCC group has evolved BHNS binaries with 3:1 mass ratio, $R_{\text{NS}} = 14\text{km}$, and $a/M_{\text{BH}} = 0.5$ for a variety of EoS, including polytropic ($\Gamma = 2$ and $\Gamma = 2.75$) and Shen [150]. They find that the NS is disrupted in one mass transfer event in every case, and the disk masses are all comparable ($0.1\text{--}0.2M_{\odot}$), although stiffer EoS do produce larger, longer-lived tidal tails.

When tidal disruption decompresses the NS matter, neutrons and protons combine to form heavy nuclei, which heats and thickens the tidal tail [135, 150]. Metzger *et al* [151] point out that, if matter from the tail remains in marginally bound orbit for around one second, even more energy may be released by r-process nucleosynthesis, perhaps enough to unbind this matter. Assuming a hot disk is sometimes formed, as in the simulations of Janka *et al* [134], the nucleosynthesis in an outflow from such a disk has been calculated by Surman *et al* [152]. They find that *r*-process nucleosynthesis does occur in such outflows. The likely BHNS merger rate and ejecta mass make it unlikely that this is the main source of these elements, however.

4. The post-merger black hole plus disk system

It appears that a massive accretion disk ($M_{\text{disk}} \sim 10^{-2} - 10^{-1}M_{\odot}$) can be produced by either a NSNS or a BHNS merger. Such disks are initially dense ($\rho \sim 10^{10} - 10^{12}\text{g cm}^{-3}$), hot ($T \sim 1 - 10\text{MeV}$), and neutron-rich ($Y_e \sim 0.1$). Angular momentum transport, most

likely driven by MRI-induced turbulence [125], causes matter to accrete into the black hole at enormous “hyperaccretion” rates of $0.1M_{\odot}\text{s}^{-1}$ to $10M_{\odot}\text{s}^{-1}$.

Steady-state models of hyperaccretion onto black holes were constructed by Popham, Woosley, and Fryer [153]. Their models were one-dimensional (axisymmetric and vertically integrated) but incorporated GR using a Kerr background spacetime. Following the Shakura-Sunyaev prescription [154], the angular momentum transport was modeled by adding a shear viscosity of strength $\nu = \alpha c_s^2 / \Omega_K$, where c_s is the sound speed, Ω_K is the Keplerian angular velocity, and α is a (unknown) dimensionless parameter. Because neutrino emission was found to be the dominant cooling process, such disks are sometimes called “neutrino dominated advection flows” (NDAFs). Other steady-state, α -viscosity NDAF models soon followed [155, 156, 157], with the most important improvement being the inclusion of neutrino opacity effects [158].

Time evolutions of NDAFs under the influence of α -viscosity have been carried in 1D [159], 2D (axisymmetry) [160, 161, 110] and 3D [109, 111]. The 2D and 3D simulations used as their initial conditions the final state of numerical BHNS merger simulations, and they treated neutrino processes via leakage schemes. Shibata, Sekiguchi, and Takahashi [162] have taken an important step by including magnetic fields in their NDAF evolutions. They evolve the MHD equations in GR but assume axisymmetry. This allows their code to capture effects related to the MRI, eliminating the need for an α viscosity.

The stresses that drive angular momentum transport also cause energy to be dissipated as heat, raising the temperature of the disk to $T \sim 10\text{MeV}$. Photons in the NDAF are trapped, so the disk cannot cool by photon radiation. In the inner $\sim 10^3\text{km}$, the disk is dense and hot enough to cool by neutrino emission. (According to most merger simulations, this would include the entire disk.) In sufficiently massive disks, for which the density reaches $\rho \geq 10^{11}\text{g cm}^{-3}$, neutrinos can become trapped in the inner region, and thermal energy is advected into the hole rather than radiated. For $\dot{M} > 1M_{\odot}\text{s}^{-1}$, neutrino trapping significantly limits the radiated energy that might power a GRB [158].

To power a GRB, some 10^{50}erg s^{-1} must be transferred from the BH-disk system to a low-mass, ultrarelativistic outflow. NDAF simulations have investigated the possibility that this energy is provided by neutrino emission. They find that the neutrino luminosity L_{ν} increases steeply with disk mass M_{disk} , viscous strength α , and corotating BH spin a/M_{BH} [153, 111, 157, 162]. In favorable circumstances ($M_{\text{disk}} \sim 0.1M_{\odot}$, $\alpha \sim 0.1$, $a/M_{\text{BH}} > 0.5$), $L_{\nu} \sim 10^{53}\text{erg s}^{-1}$ can be released, of which a few percent could be converted to a pair-photon fireball by $\nu\bar{\nu}$ annihilation [111]. This is sufficient to explain observed short GRB energies if the outflow is collimated into a $\sim 1\%$ angle. GRMHD simulations of NDAFs also find $L_{\nu} \sim 10^{53}\text{erg s}^{-1}$, indicating that the “true” effective α is around 0.01-0.1 [162]. These simulations also found strong variations in L_{ν} on millisecond timescales, in agreement with the observed variability of GRBs.

5. Future work

The needed future improvements in NSNS/BHNS merger modeling fall into three categories: improved microphysics, improved coverage of parameter space, and improved numerical accuracy. For GRB modeling, accurate microphysics is the most pressing need. Numerical simulations aiming to determine whether or not a sufficiently energetic, sufficiently ultrarelativistic outflow is generated will probably need GR, MHD, and neutrino cooling, heating, and scattering. A full solution to the neutrino transport problem is currently not feasible, but we may hope that approximate treatments will capture the essential physics, such as $\nu\bar{\nu}$ energy deposition and ν -driven winds. For the other main goal of providing gravitational waveform catalogs for LIGO data analysis, covering the NSNS and BHNS parameter space is the most important issue. The ongoing studies of the effect of the equation of state on the waveforms using parameterized EoS are obviously vital. Another poorly studied variable is the BH spin in BHNS binaries. So far, only a few cases have been studied in GR, and all of them belonged to the special case of aligned spin. Numerically, one major challenge is to improve resolution so that turbulent flows are adequately treated. Another is to extend the evolution time so that more NSNS mergers can be evolved to the point of delayed collapse. Fortunately, each of these issues is being pursued by at least one research group.

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